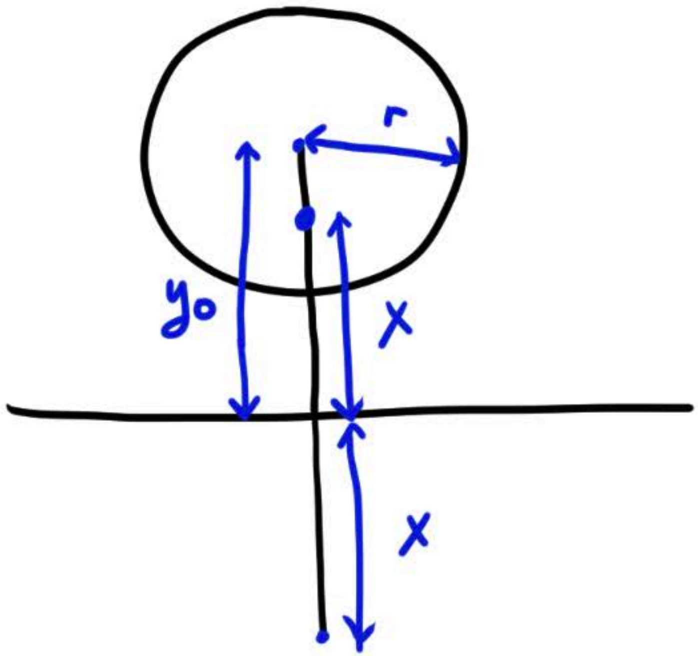
Q1



Solve $(y_0 - x)(y_0 + x) = r^2$

$$\Rightarrow \quad x = \sqrt{y_0^2 - r^2}$$

Q2 $\quad Tz = \dfrac{4iz}{z+2i}$, $\quad Sz = \dfrac{(2-i)z+1+i}{(1+i)z+i}$

a) fixed pt(s) :

$T: \quad 0, 2i$

$S: \quad -i$

b) T: Consider $z \mapsto \frac{z}{z-2i}$

which sends $0 \to 0$

$2i \to +\infty$

$$\Rightarrow \quad \frac{Tz}{Tz-2i} = \lambda \frac{z}{z-2i} \quad \text{for some } \lambda \in \mathbb{C}^\times$$

Put $z = \infty$, get $\boxed{\lambda = 2}$

S: Consider $z \mapsto \frac{1}{z+i}$

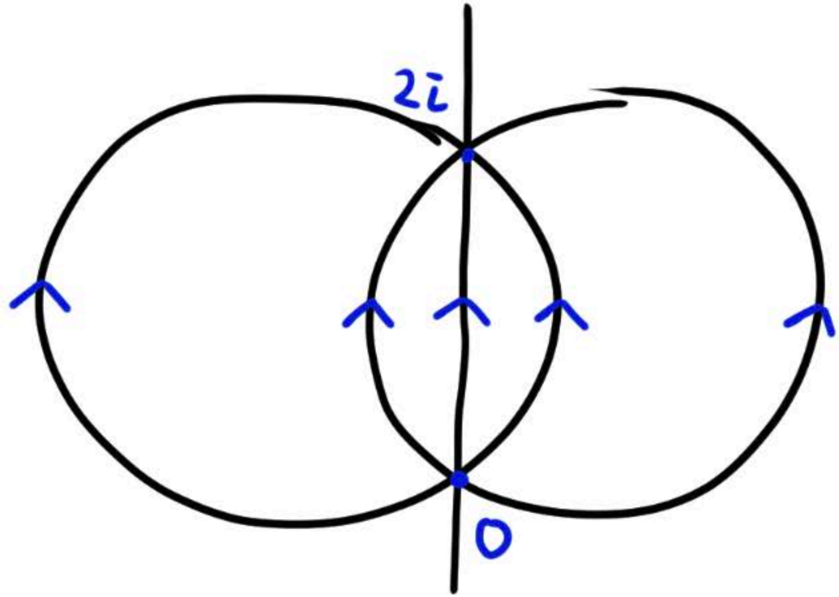
which sends $-i \to \infty$

$$\Rightarrow \quad \frac{1}{Sz+i} = \frac{1}{z+i} + \beta \quad \text{for some } \beta \in \mathbb{C}$$

Put $z = \infty$, get $\boxed{\beta = 1+i}$

c)

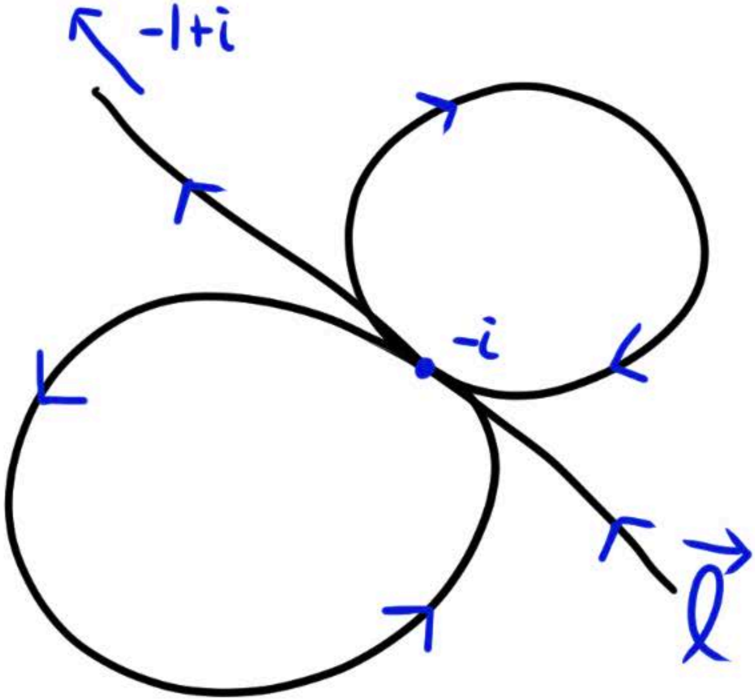
T:



Since $\lambda = 2 > 1$, T is "expanding"

$\begin{pmatrix} 0 \longrightarrow \infty & w\text{-plane} \\ 0 \longrightarrow 2i & z\text{-plane} \end{pmatrix}$

S:



$$\vec{\ell} \xleftrightarrow{z \mapsto \frac{1}{z+i}} (1+i)t \quad ; \quad \text{arrow} \longleftrightarrow t=0 \text{ to } t=+\infty$$

So $\vec{l} = -i + \dfrac{1}{(1+i)t}$ ← t from 0 to $+\infty$

$= -i + \dfrac{1-i}{2}\left(\dfrac{1}{t}\right)$ ← $\frac{1}{t}$ from $+\infty$ to 0

$= -i + (-1+i)\left(\dfrac{-1}{2t}\right)$ ← $\frac{-1}{2t}$ from $-\infty$ to 0

Q3 $\mathrm{Sim}_\infty = \{ T \in M \mid T(\infty) = \infty \}$

b) $az+b$, $a \in \mathbb{C}^\times$, $b \in \mathbb{C}$

c) Use FTMG (always $\infty \to \infty$)

d) $z \mapsto 2z \in \mathrm{Sim}_\infty \setminus$ Euclidean

e) $\#$ fixed pts $= 0$ or 1

(can't be 2 as $\exists 1$ for ∞)

f) 0: $\Rightarrow$ translation

1: $\Rightarrow$ $z \mapsto az$ (say fixed pt $= 0$)